7 Integralrechnung – Vermischte Übungen

1. Notieren Sie zuerst zu jedem der unten angegebenen Integrale eine Vermutung, mit welcher Methode es sich berechnen lässt. (Manchmal sind mehrere Methoden nacheinander anzuwenden!)
2. Berechnen Sie die bestimmten Integrale.
3. Bestimmen Sie bei denjenigen Aufgaben, die Sie spannend finden, zusätzlich das unbestimmte Integral.

		Vermutung:
a)	$\int_0^{0,5} \frac{9x+3}{x^2-1}\,dx$	Partialbruchzerlegung
b)	$\int_0^{9} \frac{\sqrt{x}}{4+\sqrt{x}}\,dx$	Substitution der Integrationsvariable
c)	$\int_{-\pi}^{\pi} (x+3)\cdot\sin(x)\,dx$	Partielle Integration
d)	$\int_0^{\ln 5} \frac{e^x-1}{e^x+1}\,dx$	Substitution der Integrationsvariable, Partialbruchzerlegung
e)	$\int_{-1}^{1} \cos^2(\pi x)\,dx$	Partielle Integration mit trigonometrischem Pythagoras
f)	$\int_{0,75}^{1} \frac{5}{(4x-5)^4}\,dx$	Lineare Substitution
g)	$\int_0^{1} x\cdot\sin(x^2)\,dx$	Substitution
h)	$\int_2^{3} \frac{x^2}{x-1}\,dx$	Polynomdivision

Lösungen

a) $\int_0^{0,5} \frac{9x+3}{x^2-1}\,dx = \int_0^{0,5} \frac{3}{x+1}+\frac{6}{x-1}\,dx = 3\ln 3 - 9\ln 2$

$$\int \frac{9x+3}{x^2-1}\,dx = \int \frac{3}{x+1}+\frac{6}{x-1}\,dx = 3\ln|x+1|+6\ln|x-1|$$

b) $\int_0^{9} \frac{\sqrt{x}}{4+\sqrt{x}}\,dx = \int_4^{7} \frac{2(u-4)^2}{u}\,du = \int_4^{7} 2u-16+\frac{32}{u}\,du = -15+32\ln 7-64\ln 2$

mit der Substitution $u=4+\sqrt{x}$

$$\int \frac{\sqrt{x}}{4+\sqrt{x}}dx = \int 2u-16+\frac{32}{u}du = u^2-16u+32\ln|u| = \left(4+\sqrt{x}\right)^2-16\left(4+\sqrt{x}\right)+32\ln\left(4+\sqrt{x}\right)$$

$$= x-8\sqrt{x}-48+32\ln\left(4+\sqrt{x}\right)$$

c) $$\int_{-\pi}^{\pi}(x+3)\cdot\sin(x)dx = \left[(x+3)\cdot(-\cos(x))\right]_{-\pi}^{\pi}-\int_{-\pi}^{\pi}1\cdot(-\cos(x))dx$$

$$= (\pi+3)-(-\pi+3)+\left[\sin(x)\right]_{-\pi}^{\pi} = 2\pi$$

$$\int(x+3)\cdot\sin(x)dx = -(x+3)\cdot\cos(x)+\sin(x)$$

d) $$\int_0^{\ln 5}\frac{e^x-1}{e^x+1}dx = \int_2^6\frac{u-2}{u(u-1)}du = \int_2^6\frac{2}{u}-\frac{1}{u-1}du = 2\ln 3-\ln 5$$ mit der Subst. $u=e^x+1$

$$\int\frac{e^x-1}{e^x+1}dx = 2\ln|u|-\ln|u-1| = 2\ln\left(e^x+1\right)-\ln\left(e^x\right) = \ln\left(e^x+1\right)^2-x$$

e) $$\underbrace{\int_{-1}^{1}\cos^2(\pi x)dx}_{=A} = \left[\cos(\pi x)\cdot\frac{1}{\pi}\sin(\pi x)\right]_{-1}^{1}-\int_{-1}^{1}-\sin^2(\pi x)dx = \int_{-1}^{1}1-\cos^2(\pi x)dx = 2-\underbrace{\int_{-1}^{1}\cos^2(\pi x)dx}_{=A}$$

$$A=2-A \quad\Rightarrow\quad A=1$$

$$\int\cos^2(\pi x)dx = \frac{1}{\pi}\cos(\pi x)\cdot\sin(\pi x)+\int\sin^2(\pi x)dx$$

$$\underbrace{\int\cos^2(\pi x)dx}_{=A} = \frac{1}{\pi}\cos(\pi x)\sin(\pi x)+x-\underbrace{\int\cos^2(\pi x)dx}_{=A} \quad |+A\ |:2$$

$$\int\cos^2(\pi x)dx = \frac{1}{2\pi}\cos(\pi x)\sin(\pi x)+\frac{1}{2}x$$

f) $$\int_{0,75}^{1}\frac{5}{(4x-5)^4}dx = \left[\frac{5}{4}\cdot\frac{1}{-3}\cdot(4x-5)^{-3}\right]_{0,75}^{1} = \frac{35}{96}$$

$$\int\frac{5}{(4x-5)^4}dx = \frac{5}{12(4x-5)^3}$$

g) $$\int_0^1 x\cdot\sin\left(x^2\right)dx = \int_0^1\sin(u)\cdot\frac{1}{2}du = \frac{1}{2}-\frac{1}{2}\cos(1)$$

$$\int x\cdot\sin\left(x^2\right)dx = -\frac{1}{2}\cos(u) = -\frac{1}{2}\cos\left(x^2\right)$$

h) $$\int_2^3\frac{x^2}{x-1}dx = \int_2^3 x+1+\frac{1}{x-1}dx = \left[\frac{1}{2}x^2+x+\ln|x-1|\right]_2^3 = 3{,}5+\ln 2$$

$$\int\frac{x^2}{x-1}dx = \frac{1}{2}x^2+x+\ln|x-1|$$