Bestimmen von Stammfunktionen

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Aufgaben:

1a) $\int \frac{x}{\sqrt{4-x^2}}\,dx$

Subst.: $u = 4-x^2$ (1)

$\frac{du}{dx} = -2x \quad |\cdot(-\frac{1}{2})\,dx$

$-\frac{1}{2}du = x\,dx$

$\overset{(1)}{=} \int -\frac{1}{2\sqrt{u}}\,du = -\sqrt{u}$

$\overset{(1)}{=} -\sqrt{4-x^2} = F(x)$

$F'(x) = -\frac{1}{2}(4-x^2)^{-\frac{1}{2}}\cdot(-2x) = \frac{x}{\sqrt{4-x^2}}$

b) $\int \sqrt{4-2x}\,dx$

Subst.: $u = 4-2x$ (1)

$\frac{du}{dx} = -2$

$-\frac{1}{2}du = dx$

$\overset{(1)}{=} \int -\frac{1}{2}\sqrt{u}\,du = -\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}}$

$= -\frac{1}{3}u^{\frac{3}{2}} \overset{(1)}{=} -\frac{1}{3}(4-2x)^{\frac{3}{2}} = F(x)$

$F'(x) = -\frac{1}{3}\cdot\frac{3}{2}(4-2x)^{\frac{1}{2}}\cdot(-2) = \sqrt{4-2x}$

2.a) $\int \frac{e^{2x}}{(e^x-2)^3}\,dx$

Subst.: $u = e^x - 2$ (1)

$x = \ln(u+2)$ (2)

$(u>0)$

$\frac{dx}{du} = \frac{1}{u+2}$

$dx = \frac{1}{u+2}\,du$

$\overset{(1),(2)}{=} \int \frac{e^{2\ln(u+2)}}{u^3}\cdot\frac{1}{u+2}\,du$

$= \int \frac{(u+2)^2}{u^3}\cdot\frac{1}{u+2}\,du$

$= \int \frac{u+2}{u^3}\,du = \int \frac{1}{u^2} + \frac{2}{u^3}\,du$

$= -\frac{1}{u} - \frac{2}{2u^2} \overset{(1)}{=} -\frac{1}{e^x-2} - \frac{1}{(e^x-2)^2}$

$= \frac{-e^x+1}{(e^x-2)^2} = F(x)$

$$F'(x) = \frac{-e^x(e^x-2)^2 - (-e^x+1)\cdot 2(e^x-2)\cdot e^x}{(e^x-2)^4}$$ (Quotientenregel)

$$= \frac{(e^x-2)\cdot[-e^{2x}+2e^x+2e^{2x}-2e^x]}{(e^x-2)^4}$$

$$= \frac{e^{2x}}{(e^x-2)^3}$$

b) $\int \frac{1}{\sqrt{x}+x}\,dx = \int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx$

Subst.: $u = 1+\sqrt{x}$ (1)

$\sqrt{x} = u-1$ (2)

$x = (u-1)^2 \quad (u \geq 1)$

$\frac{dx}{du} = 2(u-1)$

$dx = 2(u-1)\,du$

$$\overset{(1),(2)}{=} \int \frac{1}{(u-1)\cdot u}\cdot 2(u-1)\,du$$

$$= \int \frac{2}{u}\,du = 2\ln(u)$$

$$\overset{(1)}{=} 2\ln(1+\sqrt{x}) = F(x)$$

$$F'(x) = \frac{2}{1+\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{(1+\sqrt{x})\sqrt{x}} = \frac{1}{\sqrt{x}+x}$$

3a) $\int \underbrace{2x}_{v}\,\underbrace{e^{3x}}_{u'}\,dx = \frac{1}{3}e^{3x}\cdot 2x - \int 2\cdot\frac{1}{3}e^{3x}\,dx$

$$= \frac{2}{3}xe^{3x} - \frac{2}{9}e^{3x} = F(x)$$

$$F'(x) = \frac{2}{3}\cdot(e^{3x}+x\cdot 3e^{3x}) - \frac{2}{3}e^{3x} = 2xe^{3x}$$

b) $\int x\cdot\sin(x)\,dx = -x\cdot\cos(x) + \int\cos(x)\,dx$

$$= -x\cdot\cos(x) + \sin(x) = F(x)$$

$$F'(x) = -\cos(x) + x\cdot\sin(x) + \cos(x) = x\cdot\sin(x)$$

c) $\underbrace{\int e^{2x}\cdot\sin(x)\,dx}_{=F(x)} = -e^{2x}\cdot\cos(x) + \int 2e^{2x}\cos(x)\,dx$

$$= -e^{2x}\cdot\cos(x) + 2e^{2x}\sin(x) - \underbrace{\int 4e^{2x}\sin(x)\,dx}_{=4F(x)}$$

$\Rightarrow F(x) = -e^{2x}\cos(x) + 2e^{2x}\sin(x) - 4A \quad | +4F(x) \quad | :5$

$$F(x) = \frac{e^{2x}}{5}\left(-\cos(x) + 2\sin(x)\right)$$

$$F'(x) = \frac{2e^{2x}}{5}\left(-\cos(x) + 2\sin(x)\right) + \frac{e^{2x}}{5} \cdot \left(\sin(x) + 2\cos(x)\right)$$

$$= \left(\frac{4}{5} + \frac{1}{5}\right) e^{2x} \sin(x) = e^{2x} \cdot \sin(x)$$