**Vertiefungskurs Mathematik 12**

**Lösungen: Aufgaben zu komplexen Potenzen und Wurzeln**

**AUFGABE 1**

a) $-5+12i$ b) $-117-44i$ c) $8-8i$  d) $-524288+524288i$

**AUFGABE 2**

a) $e^{\frac{5}{4}πi}$ b) $e^{6πi}=1$ c) $e^{\frac{6}{5}πi}$ d) $e^{\frac{1}{10}πi}$ e) $e^{14i}≈e^{1,344i}$

**AUFGABE 3**

a) $64e^{\frac{6}{5}πi}$ b) $\frac{1}{128}e^{\frac{5}{4}πi}$ c) $-512$ d) $8e^{\frac{2}{5}πi}$ e) $625e^{2,966i}$

**AUFGABE 4**

a) $n=8$b) $n=40$ c) $n=10$

**AUFGABE 5**

a) $z\_{1}=e^{\frac{1}{2}πi}$ ; $z\_{2}=e^{\left(\frac{1}{2}+\frac{2}{3}\right)πi}=e^{\frac{7}{6}πi}$ ; $z\_{3}=e^{\left(\frac{1}{2}+2∙\frac{2}{3}\right)πi}=e^{\frac{11}{6}πi}$

b) $z\_{1}=e^{\frac{1}{6}πi}$ ; $z\_{2}=e^{\left(\frac{1}{6}+\frac{1}{2}\right)πi}=e^{\frac{2}{3}πi}$ ; $z\_{3}=e^{\left(\frac{1}{6}+2∙\frac{1}{2}\right)πi}=e^{\frac{7}{6}πi}$ ; $z\_{4}=e^{\left(\frac{1}{6}+3∙\frac{1}{2}\right)πi}=e^{\frac{5}{3}πi}$

c) $z\_{1}=e^{\frac{1}{12}πi}$ ; $z\_{2}=e^{\left(\frac{1}{12}+\frac{1}{3}\right)πi}=e^{\frac{5}{12}πi}$ ; $z\_{3}=e^{\left(\frac{1}{12}+2∙\frac{1}{3}\right)πi}=e^{\frac{3}{4}πi}$

 $z\_{4}=e^{\left(\frac{1}{12}+3∙\frac{1}{3}\right)πi}=e^{\frac{13}{12}πi}$ ; $z\_{5}=e^{\left(\frac{1}{12}+4∙\frac{1}{3}\right)πi}=e^{\frac{17}{12}πi}$ ; $z\_{6}=e^{\left(\frac{1}{12}+5∙\frac{1}{3}\right)πi}=e^{\frac{7}{4}πi}$

d) $z\_{1}=e^{\frac{1}{40}πi}$ ; $z\_{2}=e^{\left(\frac{1}{40}+\frac{1}{4}\right)πi}=e^{\frac{11}{40}πi}$ ; $z\_{3}=e^{\left(\frac{1}{40}+2∙\frac{1}{4}\right)πi}=e^{\frac{21}{40}πi}$

 $z\_{4}=e^{\left(\frac{1}{40}+3∙\frac{1}{4}\right)πi}=e^{\frac{31}{40}πi}$ ; $z\_{5}=e^{\left(\frac{1}{40}+4∙\frac{1}{4}\right)πi}=e^{\frac{41}{40}πi}$ ; ; $z\_{6}=e^{\left(\frac{1}{40}+5∙\frac{1}{4}\right)πi}=e^{\frac{51}{40}πi}$

 $z\_{7}=e^{\left(\frac{1}{40}+6∙\frac{1}{4}\right)πi}=e^{\frac{61}{40}πi}$ ; ; $z\_{8}=e^{\left(\frac{1}{40}+7∙\frac{1}{4}\right)πi}=e^{\frac{71}{40}πi}$

e) $z\_{1}=e^{0,13i}$ ; $z\_{2}=e^{\left(0,13+\frac{2}{9}π\right)i}≈e^{0,828i}$ ; $z\_{3}=e^{\left(0,13+2∙\frac{2}{9}π\right)i}≈e^{1,526i}$

 $z\_{4}=e^{\left(0,13+3∙\frac{2}{9}π\right)i}≈e^{2,224i}$ ; $z\_{5}=e^{\left(0,13+4∙\frac{2}{9}π\right)i}≈e^{2,923i}$

 $z\_{6}=e^{\left(0,13+5∙\frac{2}{9}π\right)i}≈e^{3,621i}$ ; $z\_{7}=e^{\left(0,13+6∙\frac{2}{9}π\right)i}≈e^{4,319i}$

 $z\_{8}=e^{\left(0,13+7∙\frac{2}{9}π\right)i}≈e^{5,017i}$ ; $z\_{9}=e^{\left(0,13+8∙\frac{2}{9}π\right)i}≈e^{5,715i}$

**AUFGABE 6**

a) $z\_{1}=2e^{\frac{1}{4}πi}$ ; $z\_{2}=2e^{\left(\frac{1}{4}+\frac{2}{3}\right)πi}=2e^{\frac{11}{12}πi}$ ; $z\_{3}=2e^{\left(\frac{1}{4}+2∙\frac{2}{3}\right)πi}=2e^{\frac{19}{12}πi}$

b) $z\_{1}=\frac{1}{2}e^{\frac{1}{10}πi}$ ; $z\_{2}=\frac{1}{2}e^{\left(\frac{1}{10}+\frac{1}{2}\right)πi}=\frac{1}{2}e^{\frac{3}{5}πi}$ ; $z\_{3}=\frac{1}{2}e^{\left(\frac{1}{10}+2∙\frac{1}{2}\right)πi}=\frac{1}{2}e^{\frac{11}{10}πi}$

 $z\_{4}=\frac{1}{2}e^{\left(\frac{1}{10}+3∙\frac{1}{2}\right)πi}=\frac{1}{2}e^{\frac{8}{5}πi}$

c) $z\_{1}=\sqrt{2}e^{\frac{1}{12}πi}$ ; $z\_{2}=\sqrt{2}e^{\left(\frac{1}{12}+\frac{1}{4}\right)πi}=\sqrt{2}e^{\frac{1}{3}πi}$ ; $z\_{3}=\sqrt{2}e^{\left(\frac{1}{12}+2∙\frac{1}{4}\right)πi}=\sqrt{2}e^{\frac{7}{12}πi}$

 $z\_{4}=\sqrt{2}e^{\left(\frac{1}{12}+3∙\frac{1}{4}\right)πi}=\sqrt{2}e^{\frac{5}{6}πi}$ ; $z\_{5}=\sqrt{2}e^{\left(\frac{1}{12}+4∙\frac{1}{4}\right)πi}=\sqrt{2}e^{\frac{13}{12}πi}$

 $z\_{6}=\sqrt{2}e^{\left(\frac{1}{12}+5∙\frac{1}{4}\right)πi}=\sqrt{2}e^{\frac{4}{3}πi}$ ; $z\_{7}=\sqrt{2}e^{\left(\frac{1}{12}+6∙\frac{1}{4}\right)πi}=\sqrt{2}e^{\frac{19}{12}πi}$

 $z\_{8}=\sqrt{2}e^{\left(\frac{1}{12}+7∙\frac{1}{4}\right)πi}=\sqrt{2}e^{\frac{11}{6}πi}$

d) $z\_{1}=2e^{\frac{6}{25}πi}$ ; $z\_{2}=2e^{\left(\frac{6}{25}+\frac{2}{5}\right)πi}=2e^{\frac{16}{25}πi}$ ; $z\_{3}=2e^{\left(\frac{6}{25}+2∙\frac{2}{5}\right)πi}=2e^{\frac{26}{25}πi}$

 $z\_{4}=2e^{\left(\frac{6}{25}+3∙\frac{2}{5}\right)πi}=2e^{\frac{36}{25}πi}$ ; $z\_{5}=2e^{\left(\frac{6}{25}+4∙\frac{2}{5}\right)πi}=2e^{\frac{46}{25}πi}$

e) $z\_{1}=\sqrt{3}e^{0,3πi}$ ; $z\_{2}=\sqrt{3}e^{\left(0,3+\frac{1}{3}π\right)i}≈\sqrt{3}e^{1,347πi}$ ; $z\_{3}=\sqrt{3}e^{\left(0,3+2∙\frac{1}{3}π\right)i}≈\sqrt{3}e^{2,394i}$

 $z\_{4}=\sqrt{3}e^{\left(0,3+3∙\frac{1}{3}π\right)i}≈\sqrt{3}e^{3,442i}$ ; $z\_{5}=\sqrt{3}e^{\left(0,3+4∙\frac{1}{3}π\right)i}≈\sqrt{3}e^{4,489i}$

 $z\_{6}=\sqrt{3}e^{\left(0,3+5∙\frac{1}{3}π\right)i}≈\sqrt{3}e^{5,536i}$