**Implikation und Äquivalenz – Lösungen**

**1.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| a) | A | B | ¬B | **A → (¬B)** |
|  | w | w | f | f |
|  | w | f | w | w |
|  | f | w | f | w |
|  | f | f | w | w |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| b) | A | B | B → A | **A ∧ (B → A)** |
|  | w | w | w | w |
|  | w | f | w | w |
|  | f | w | f | f |
|  | f | f | w | f |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| c) | A | B | ¬B | **A → (¬B)** |
|  | w | w | f | f |
|  | w | f | w | w |
|  | f | w | f | w |
|  | f | f | w | w |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| d) | A | B | A ∧ B | **A → (A ∧ B)** |
|  | w | w | w | w |
|  | w | f | f | f |
|  | f | w | f | w |
|  | f | f | f | w |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| e) | A | B | C | B ∧ C | **A → (B ∧ C)** |
|  | w | w | w | w | w |
|  | w | w | f | f | f |
|  | w | f | w | f | f |
|  | w | f | f | f | f |
|  | f | w | w | w | w |
|  | f | w | f | f | w |
|  | f | f | w | f | w |
|  | f | f | f | f | w |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| f) | A | B | C | B ∨ C | **A → (B ∨ C)** |
|  | w | w | w | w | w |
|  | w | w | f | w | w |
|  | w | f | w | w | w |
|  | w | f | f | f | f |
|  | f | w | w | w | w |
|  | f | w | f | w | w |
|  | f | f | w | w | w |
|  | f | f | f | f | w |

**2.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| A | B | **A → B** | ¬A | ¬B | **¬ B → ¬A** |
| w | w | w | f | f | w |
| w | f | f | f | w | f |
| f | w | w | w | f | w |
| f | f | w | w | w | w |

Die dritte und die letzte Spalte sind gleich. Damit sind A → B und ¬B → ¬A logisch äquivalent. Somit sind „Wenn es regnet, dann ist die Straße nass.“ logisch äquivalent zu „Wenn die Straße nicht nass ist, dann regnet es nicht.“ Und „Wenn Fifi ein Dackel ist, dann ist er ein Hund.“ ist logisch äquivalent zu „Wenn Fifi kein Hund ist, dann ist er kein Dackel.“

**3.** a) A ↔ B ist logisch äquivalent zu **(A → B) ∧ (B → A)**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| b) | A | B | A ↔ B | A → B | B → A | **(A → B) ∧ (B → A)** |
|  | w | w | w | w | w | w |
|  | w | f | f | f | w | f |
|  | f | w | f | w | f | f |
|  | f | f | w | w | w | w |

**4.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | A | B | A ↔ B | **¬(A ↔ B)** |
|  | w | w | w | f |
|  | w | f | f | w |
|  | f | w | f | w |
|  | f | f | w | f |

Logisch äquivalent dazu ist das „Entweder … oder“: (A ∧ (¬B)) ∨ ((¬A) ∧ B)

**5.** Es gibt 24 = 16solche Verknüpfungen, denn für jede der vier Zellen in der letzten Spalte hat man zwei Möglichkeiten (w und f).

**6.**

a) A → B b) (¬B) ∨ A c) ¬(A ↔ B) d) (¬A) ∨ (¬ B)